

NOTES ON FORMULAS FOR USE IN FORECASTING MINIMUM TEMPERATURE

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USE OF EQUATION OF HYPERBOLA IN FORECASTING FROM
HYGROMETRIC DATA

An examination of several dot-charts showing the relation between relative humidity and departure of ensuing minimum temperature from the dew-point leads me to the conclusion that, in many cases at least, the equation of a hyperbola expresses the relation even better than does the parabolic formula often used. It is proposed herein to use the rectangular hyperbola with asymptotes parallel to the axes of coordinates of the dot-chart, having, therefore, the general equation,

$$(X-a)(Y-b)=K$$

where the point (a, b) is the intersection of asymptotes and K is a constant depending upon the form of the curve. For use in forecasting minimum temperatures, this equation is to be solved for Y , giving us

$$Y = \frac{K}{X-a} + b.$$

The values of the parameters, a , b , and K , for any particular chart can be computed easily by solving three equations obtained by substituting in our general equation three pairs of values of X and Y taken for three selected points along the curve best fitting the plotted data (relative humidity as abscissas, departures of minimum temperature from dew-point as ordinates), drawn free-hand. The method of procedure is now illustrated:

Figure 1 is a chart of this kind prepared from data for San Jose, California, for March during the period, 1914 to 1925, inclusive, considering cases with clear skies at observation time in the morning. The points chosen for computing the constants of the desired equation are (15, 24), (40, -2), and (70, -10), indicated by the letters A, B, and C, respectively, on the chart. Substituting these three pairs of values in the general equation, we have

$$\begin{aligned} \text{(A). } (15-a)(24-b) &= K = 360 - 24a - 15b + ab \\ \text{(B). } (40-a)(-2-b) &= K = -80 + 2a - 40b + ab \\ \text{(C). } (70-a)(-10-b) &= K = -700 + 10a - 70b + ab \end{aligned}$$

Eliminating K , by equating its values from (A) and (B), and then those from (B) and (C), we have

$$\begin{aligned} (1). \quad 26a - 25b &= 440 \\ (2). \quad 8a - 30b &= 620 \end{aligned}$$

Eliminating a from (1) and (2), we have, $b = -21.7$

Substituting this value in (2), we get, $a = -3.9$

Substituting in (A), we obtain, $K = 863.7$

Then our desired equation is, using whole numbers for constants

$$Y = \frac{864}{X+4} - 22.$$

We may check this equation by substituting therein a value of either unknown taken from the curve, and comparing the value of the other unknown thus computed with the corresponding value on the curve. Thus, where $X = 26$ we get,

$$Y = \frac{864}{26+4} - 22 = 6.8,$$

which agrees with the value from the curve. Again, where $X = 46$ we have,

$$Y = \frac{864}{46+4} - 22 = 4.7,$$

which agrees closely with the curve.

A similar equation may be derived for Grand Junction, Colorado, from Figure 7A on page 43 of Supplement No. 16 to the Monthly Weather Review. Take the three points (10, 34), (25, 16), and (55, 0). Then we have

$$\begin{aligned} \text{(A)} \quad (10-a)(34-b) &= K = 340 - 34a - 10b + ab \\ \text{(B)} \quad (25-a)(16-b) &= K = 400 - 16a - 25b + ab \\ \text{(C)} \quad (55-a)(0-b) &= K = -55b + ab \end{aligned}$$

From which we obtain: $a = -20$, $b = -24$, $K = 1,800$, and the equation,

$$Y = \frac{1,800}{X+20} - 24.$$

When X is 40, e. g., this gives a value of 6 for Y , which agrees closely with the curve. When X is 5, Y is 48, which agrees with plotted data better than the curve drawn.

Taking Fig. 6, for El Paso, Tex., on page 11 of the same supplement: Choose the points (5, 50), (15, 25), and (35, 10), and we obtain: $a = -7.9$, $b = -7.2$, $K = 737.9$, and our equation is, using whole numbers,

$$Y = \frac{738}{X+8} - 7.$$

From this equation we compute the following pairs of values: $X, 22, Y, 17.6$; $X, 10, Y, 34$; $X, 2, Y, 66.8$. These values seem to fit the plotted data even better than the line drawn.

Again, taking Fig. 2 on page 528 of the Monthly Weather Review for October, 1922, minimum temperature prediction graph for Spokane, Wash.; choose the points (10, 30), (30, 10), (60, 0), and we have equations giving for a value, -15, for b , the same, and for K , 1125. The hyperbolic equation for this curve is, therefore,

$$Y = \frac{1,125}{X+15} - 15,$$

which agrees closely with the data entered on the chart.

ARRANGEMENT FOR CONVENIENT COMPUTATION

Our general equation may be adapted to logarithmic computation. Thus, by transposition, we have

$$Y-b = \frac{K}{X-a}.$$

Then, $\log(Y-b) = \log K - \log(X-a)$.

The following form of table may be used for convenience:

X (Rel. Hum.)					
a					
$X-a$					
$\log(X-a)$					
$\log K$					
$\log(Y-b)$					
$(Y-b)$					
b					
Y					

USE OF DOT-CHART IN FORECASTING

It is well, in actual practice, to have at hand the original dot-chart on which the graph of the derived equation has been plotted, in order that, from "variations in scattering of dots in different regions," one may note "roughly what dependence may be placed on different portions of the curve." In fact, some may prefer to avoid the labor of computing the constants of an equation and to depend upon a curve drawn free-hand, as previously suggested in Supplement No. 16, already referred to; in this case it is unimportant whether the curve be of parabolic, hyperbolic, or other form, since

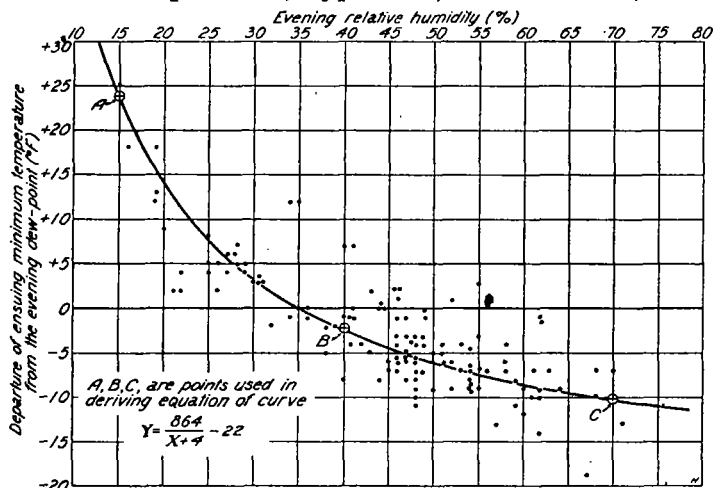


FIG. 1.—Relation between evening relative humidity and departure of ensuing minimum temperature from the evening dew point at San Jose, Calif. (Based on data for March, 1914 to 1925, inclusive, during clear weather)

results will be taken directly from the graph, without computation.

COMPARISON OF PARABOLA AND HYPERBOLA USED

In some cases, the parabola and the hyperbola appear to fit the plotted hygrometric data equally well. Sometimes, however, the parabola whose constants have been derived fits well the main body of data but departs at either or both high and low values of relative humidity. These departures are due, directly and indirectly, largely to the recurvature that takes place at the vertex of the parabola. Thus, in the case of El Paso, considered above, the upper portion of the given parabola lies below the most of the dots (at low relative humidities), while recurvature upwards takes place among the dots for the higher humidities (but still at comparatively low humidity values). But, if we change the form of the parabola to conform to data in the upper portion, recurvature becomes sharper.

In any case, taking the equation of the parabola in proper position to fit the data plotted as considered in these notes, we have, $Y = a + bX + cX^2$, where a , b , and c are constants. Then the vertex of our parabola lies at the point where X has the value $-\frac{1}{2}b/c$, and recurvature upward takes place at this point (i. e., Y increases thereafter at an increasing rate, with increase of X), as may be shown by the methods of Calculus. Thus, differentiating, we get

$$\frac{dy}{dx} = b + 2cx$$

slope of the curve. This, at the vertex is 0; then we have,

$$b + 2cx = 0; x = -\frac{b}{2c}$$

If this point be at or beyond the highest relative humidity value (X) used in forecasting, no harm is done.

This (sometimes) objectionable recurvature does not take place in our hyperbola; this curve continues downward (with decreasing angle with the X -axis), at high values of X becoming approximately horizontal.

The method employed above in deriving constants for hyperbolic equations is an adaptation of the "Star Point" method of Marvin and Smith which avoids (without sacrifice of useful accuracy, in this problem), "the tedious and laborious least square method."¹

PREDICTING MINIMUM TEMPERATURES FROM THE DEPRESSION OF THE DEW POINT BELOW THE MAXIMUM TEMPERATURE

In some cases, by considering the depression of the minimum temperature below the maximum temperature of the preceding day as a function of the excess of that maximum temperature over the dew point observed the same evening, we may obtain a chart or formula that will be useful in forecasting minimum temperatures. In the case of data obtained during the spring of 1926 at the fruit-district substation at Hollister, San Benito County, Calif., the results of this method have been

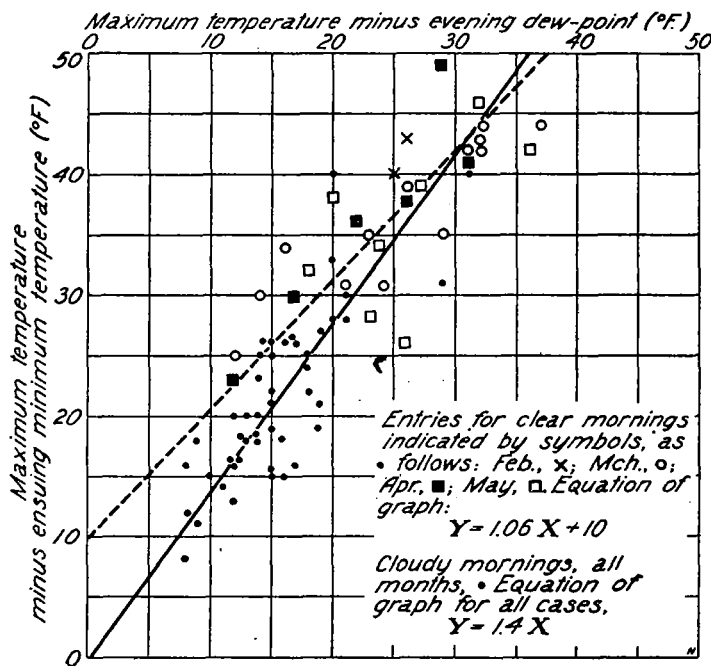


FIG. 2.—Relation between maximum temperature minus evening dew point and maximum temperature minus ensuing minimum temperature for February, March, April, and May, 1926, at Hollister, Calif. (San Jose fruit district)

more promising than those obtained otherwise. Figure No. 2 herewith is a dot chart on which have been plotted, from the Hollister data, depressions of dew points below the maximum temperatures as abscissas and depressions of minimum temperatures below the maxima as ordinates. Cases with clear mornings are indicated by special symbols, and the straight line that appears to fit these cases best has been drawn free-hand; the equation of the line is $Y = 1.06X + 10$, where Y and X are the specified depressions of minimum temperature and dew

¹ Supplement No. 16, MONTHLY WEATHER REVIEW.

point, respectively. For all cases the straight line $Y=1.4X$, has been drawn. Forecasting by this method, considering the scattering of dots from the graphs, would be equally successful in clear or cloudy weather.

It will be noted that this method is essentially a modification of my maximum-minimum temperature method (described in Monthly Weather Review Supplement No. 16 and elsewhere) by considering the effect of moisture in retarding cooling.

551.515 (73) THE TORNADO

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The tornado discussed in what follows is the typical "twister" of the American prairies, and may be defined as a slightly funnel-shaped, or flaring, hollow, circular column of upward-spiraling winds of destructive velocity. It is the most violent, least extensive, and most sharply defined of all storms. Its appearance and effects often have been described. There is no satisfactory account, however, of its origin. Hence it seems worth while to assemble the more common facts of observation in connection with this type of storm, and to deduce therefrom whatever we can in regard to its genesis. These deductions, being something definite to prove or disprove, will at least help to fix one's attention and thereby hasten, it is hoped, the ultimate solution of this difficult meteorological problem.

Some of the normal, but not all of them invariable, circumstances of place and meteorological conditions connected with the occurrence of tornadoes are the following, the more important, from the standpoint of this paper, being numbered with boldface type.

1. Geographic location.—Central and southeastern United States, chiefly; next, perhaps, southern Australia, though Griffith Taylor says, in his *Australian Meteorology*, "tornadoes are not common in Australia"; and several other parts of the world occasionally, except in general the tropical regions. The so-called tornado of tropical west Africa appears to be a violent thunderstorm of the squall type. The tropical waterspout is relatively mild, and of a more or less different origin.

2. Meteorological location.—Southeastern section, or, more exactly, east of the wind-shift line, of a Low, or cyclone, of moderate to decided intensity.

3. Kind of cyclone.—The trough or V-shaped, the kind productive of secondary cyclones, is very favorable, especially when the V protrusion points southward, or, more particularly, southwestward. However, tornadoes occur also when this protrusion of the isobars is not conspicuous, if indeed present at all, at the surface of the earth.

4. Other pressure distribution.—A moderate anticyclone to the rear, that is, west or northwest, of the cyclone, appears to be an invariable condition; but even if this pressure distribution be essential, as we believe it is, to the genesis of the tornado there is no proof of it from statistical evidence alone, since normally the extratropical cyclone has an anticyclone to its rear.

5. Surface pressure gradient in region of tornado.—Usually moderate to steep in comparison with the average cyclone.

6. Horizontal temperature gradient.—Usually steep along a portion of the border between cyclone and anticyclone.

7. Previous wind.—Moderate to fresh southerly, often southwest.

8. Following wind.—Moderate to fresh, northerly, often northwest.

9. Previous temperature.—At 8 a. m. 70° or over and increasing.

10. Following temperature.—Distinctly lower than just before the storm.

11. Previous humidity.—Excessive—making the air, at its high temperature, sultry and oppressive, from hours to even days before.

12. Clouds.—Heavy cumulo-nimbus, from which a funnel-shaped cloud depends. Sometimes this cumulus is isolated and very towering, but, when not isolated, often preceded briefly to an hour or longer by mammato-cumuli.

13. Precipitation.—Rain and usually hail 10 to 30 minutes before; light precipitation at instant of storm (funnel cloud often clearly seen and occasionally photographed); deluge of rain, mixed at times with small hail, shortly after.

14. Lightning.—Nearly, or quite, invariably lightning accompanies the tornado, but seldom, if at all, occurs in the funnel cloud.

15. Sounds.—There always is a loud rumbling or roaring noise while the whirling pendant cloud is in touch with or even closely approaches, the earth.

16. Direction of tornado wind.—Spirally upward around a traveling axis, and in the same sense as the accompanying cyclone—counterclockwise in the northern hemisphere.

17. Horizontal velocity of wind in tornado.—Unmeasured, but destructively great.

18. Vertical velocity of wind in tornado.—Also unmeasured, but sufficient to carry up pieces of lumber and other objects of considerable weight—say 100 to 200 miles per hour.

19. Location of initial and sustaining whirl.—Above, probably close above, the general cloud base.

20. Velocity of storm travel.—Usually 25 to 40 miles per hour.

21. Length of path.—Anything up to possibly 300 miles, usually 20 to 40 miles.

22. Direction of travel.—Roughly parallel to travel of the center of the general or cyclonic storm, hence usually northeastward.

23. Width of storm.—Anything from 40 to 50 feet up to, rarely, a mile or even more, but averaging around 1000 feet. Many are only 500 to 600 feet across and others, as stated, even much less.

24. Number.—Usually several, often in groups, in connection with the same low-pressure system, and on the same day.

25. Time of year.—Mainly spring, and early to mid-summer, but occasionally also at other seasons.

26. Time of day.—Usually midafternoon, or 3:00 to 5:00 p. m.

All the foregoing meteorological conditions are inferred from observations at the surface of the earth, and not in the free air one or two kilometers above the surface, where the tornado seems to have its origin. Data from this obviously desirable upper level appear to be very scanty. However, through the kind assistance of the Climatological and the Aerological divisions of the United States Weather Bureau, 26 cases were found where observations by sounding balloon or kite, or both, were made less, to much less, than six hours from the time of and nearer—some far closer—than 100 miles from, a tornado. These observations indicate (they are too few to prove anything) that when tornadoes occur the wind, whatever its value at the surface, is strong (around 20 to 25 meters